Distributed Control of Multi-Agent Systems with Communication Delay for Cooperative Source Seeking

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Received 09 Apr. 2022 Received in revised form 20 May 2022 Accepted 30 June 2022

Type of Article: Research paper

***Abstract*—This paper investigates the problem of cooperative source seeking by networked multi-agent systems subject to communication time-delay; where the robots of the group are equipped with the sensors that measure the field concentration. First, a novel distributed control algorithm is proposed based on the estimation of the field gradient to move the agents along the gradient direction toward the source. Sufficient conditions are derived in terms of linear matrix inequalities to tune the parameters of the control law to assure the convergence of the team to a neighborhood of the source in spite of the estimation errors and communication time-delay. A cooperative estimation scheme is developed to obtain the gradient of the formation center by the delayed information. Finally, simulation results in Matlab® and Webots® are presented to demonstrate the efficiency and applicability of the suggested theory.**

***Keywords*—communication time-delay, distributed control, gradient estimation, source seeking.**

I. Introduction

MULTI-AGENT systems has been attracted a lot of attention thanks to their notable benefits to accomplish complicated missions compared to a single-agent ones. Multi-robot teams have several applications in practice mostly in dangerous and/or unpredictable situations, like monitoring, surveillance, search and rescue [1-3], and source seeking [4-8] in which several robots with sensors are derived toward a minimum or maximum of a scalar field that represent heat intensity, chemical concentration, light intensity, and so on. Source seeking by multi-robot systems in different scenarios such as finding poisonous gas leak, fire, and chemical spill is a challenging problem.

Many of source-seeking algorithms are based on animals' behaviors. For instance, Couzin Group showed in [9] that fish herds can do gradient tracking in complicated light environments to find darker (shaded) spots even when the field changes in time. In addition, each fish has a very low or no gradient estimate. Basically, they can measure the light intensity and react to the position of other fishes in their sight. Using these measurements, a fish in the group increases its swimming speed when the intensity of light is relatively high and they swim slower when the light intensity decreases. Through this, the group can align its path with gradient direction and swim toward shades [9].

To realize the cooperative control protocol in a multi-robot team, the data is exchanged between the agents over a communication medium. Time-delay is ubiquitous in large scale and wireless communication networks. Although several papers have been published on source seeking problem, there is no study on the control of source seeking in the presence of time delays. In [10], a scalar gradient extremum seeking controller was designed and analyzed in the presence of arbitrarily long input-output delays, using a predictor which estimate the Hessian by a perturbation-based approach. In [11], an approach was developed for multi agent systems with input delays and uncertain disturbances, that can reach the fixed-time consensus for any initial condition by an event triggered control. To achieve the fixed-time formation tracking in networked multi agent systems, a distributed sliding control manifold was presented in [12]; where, using repulsive potential function the collision free tracking was guaranteed. To

[DOI: 10.52547/jocee.1.1.85]

find a network optimizer, a control protocol was introduced in [13], which can solve distributed optimization problem using several agents with communication delay and constraints set. In [14], a solution was developed for multivariable extremum seeking problem in delayed systems via constructing a predictor using perturbation-based estimates of the model. Multiple extrema seeking algorithm as a novel mission control strategy was presented in [15] for exploring a scalar field. A solution for locating different extrema under the assumptions of a bounded search area and estimated number of extrema was found using a combination of Glowworm swarm optimization and formation control in [15].

In [16], a consensus-type coordination algorithm was proposed using the distributed estimation of the sum of gradients of individual cost functions. This algorithm can address unknown directed communication topologies in the networked system using only limited communications and information exchanges among nodes. The consensus problem of second-order multi-robot systems was investigated in [17] using the frequency domain analysis under noise conditions and various delay. The characteristic equation was transformed into the quadratic polynomial of the pure imaginary eigenvalue, and then the conditions for achieving consensus was obtained.

In [18] the condition of piecewise constant utilities was relaxed and the ultimate bound and the input-to-state stability conditions for the spacing error and its dynamics was provided. In addition, using both the frequency domain method and the Lambert-W function, the maximum allowable time delay within which the circumnavigation remains stable was derived. In [19], a method was proposed for unicycle vehicles to track a mobile target position in a circular formation. An autonomous stable ecosystem was utilized for generating the circular trajectories.

In this paper, a source seeking control algorithm is suggested for a multi-robot system in which all the members of the group collect and share their measurements of field concentration, position, and velocity via data-delaying wireless communication network. The main novelties of the paper are threefold: First, a control law which uses delayed information and gradient estimations is developed to drive the agents toward the source location with a desired formation. Second, a new Lyapunov-Krasovskii functional is employed to achieve sufficient conditions based on LMIs to adjust the control parameters to assure the convergence of the team to a neighborhood of the source. Third, a distributed gradient estimator based on delayed information is developed.

The rest of paper is organized as follows. The problem is formulated in Section 2. In Section 3, the control method is presented and then, using a Lyapunov-Krasovskii functional, the ultimate boundedness of the convergence error is analyzed considering communication time-delay. In section 4, a cooperative gradient estimation method for the multi-agent system is proposed using the distributed gradient optimization. Section 5 presents simulation results, where the real-world applicability of the introduced scheme is illustrated in the Webots®. Finally, Section 6 represents the conclusion and new directions for future research.

II. PROBLEM FORMULATION

The dynamic of the i 'th agent in the team is explained by a double integrator as follows:

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= u_i \quad for \quad i = 1, 2, ..., N \end{aligned} \tag{1}$$

where N is the number of the members of the group. x_i, v_i and $u_i \in R^n$ represent the position vector , the velocity vector, and the control input vector of the i 'th robot respectively.

Remark 1. The double integrator is a simplified model of a real robot which is described by a Lagrangian system. A Lagrangian system can be converted into a double integrator model using feedback linearization [20]. Therefore, our analysis relies on the double integrator dynamic.

The agents in the group exchange information with each other through a data-delaying wireless medium. Thus, each agent receives the data from the others with a known transmission delay τ . The connections between the agents are modeled using a directed weighted graph G in which, each agent is represented by a node and the weight on the information link (i, j) is defined as the element a_{ij} of the adjacent matrix. The Laplacian matrix L is defined as $L = D - A$ where D is in-degree matrix of graph G and is given as $D = diag\{d_1, d_2, ..., d_N\}$, and also $d_i = \sum_{j=1}^{N} a_{ij}$ is the membership degree of the node i .The following assumptions are made on the environment and source:

Assumption 1. Location of the source is fixed and the agents can measure the information needed to estimate the field gradient.

Assumption 2. The scalar value of the field is modeled by a deterministic spatial distribution, $F(x): R^n \to R$ which is a differentiable, deterministic, time-invariant, and strongly convex function with regard to

[DOI: 10.52547/jocee.1.1.85]

x and achieves its maximum at the source position $x = x_s$. Namely, the field $F(x)$ is formed by a single source, for instance a magnetic field, possesses the maximum value at x_s and decreases with an increase in its distance from the source. Note that since $F(x)$ is dependent on the position, the i th robot senses different value of the field concentration namely, $F(x_i)$.

Assumption 3. The Hessian matrix of $F\left(x\left(t\right)\right)$ for all $x\left(t\right)$ in the domain, satisfies:

$$-\kappa_1 \le \lambda_{\min}\left(H\right), \lambda_{\max}\left(H\right) \le -\kappa_2 \qquad (2)$$

with $\kappa_1 > \kappa_2 > 0$.

Assumption 4. G is weighted-balanced and connected.

The problem of interest is to develop a source seeking control law so that the center position of the robots converges toward the source $x_s = \arg\max\left(F(x)\right)$. Before proceeding, the following Lemmas, which are employed in the derivation of the results, are recalled from the literature.

Lemma 1. Assume L as a Laplacian matrix of graph G. If G is weighted-balanced, it can be proved that $1_N^T L = 0_N^T$; moreover, if is connected there exists an G orthogonal matrix $W' \in R^{(N-1)\times(N-1)}$ such that $\left[\left(1/\sqrt{N}\right)1_N, W'\right]^T L \left[\left(1/\sqrt{N}\right)1_N, W'\right] = diag\left(0, J_{(N-1)\times(N-1)}\right)$, where J is created by all positive real part eigenvalues of L.

Lemma 2. For a positive matrix P, and any $y(s) \in L_2[-\tau, 0]$, the following inequality holds.

$$\int_{-\tau}^{0} y(s)^T Py(s)ds \ge \frac{1}{\tau}\left(\int_{-\tau}^{0} y(s)^T ds\right) P \left(\int_{-\tau}^{0} y(s)ds\right).$$

Lemma 3(Schur Complements)**.** Taking into account matrices X, Y and M with compatible dimensions, $X = X^T$, $Y = Y^T$, the following LMIs are equivalent.

1) $\begin{bmatrix} X & M \\ M^T & Y \end{bmatrix} < 0$

2) Either one of the followings:

a) $Y < 0$, and $X - MY^{-1}M^T < 0$.

b) $X < 0$, and $Y - M^T X^{-1} M < 0$.

III. SOURCE SEEKING CONTROL ALGORITHEM

Regarding assumptions 1-4, we introduce the control input to the i 'th agent as follows

$$\begin{aligned} u_i(t) = {} & c_0 \hat{g}_c - c_1 v_i\left(t\right) \\ & -c_2 \sum_{j\in N(i)} a_{ij}\left(v_i\left(t-\tau\right) - v_j\left(t-\tau\right)\right) \\ & -c_3 \sum_{j\in N(i)} a_{ij}\left(\left(x_i\left(t-\tau\right) - x_{di}\right) - \left(x_j\left(t-\tau\right) - x_{dj}\right)\right). \end{aligned} \qquad (3)$$

Where, c_0, c_1, c_2, c_3 are positive scalar constants, which will be determined later. The $\hat{g}_c$ is the gradient estimation of the center $x_c = 1/N \sum_{i=1}^{N} x_i$, represents N the number of the agents.neighbor denotes the $N(i)$ set of the i 'th agent, $v_i(t)$, $x_i(t)$, $v_i(t-\tau)$,and and,velocity delayed velocity, position, are $x_i(t-\tau)$ delayed position of the i 'th robot respectively. a_{ij} denotes (i, j) 'th entry of adjacent matrix, and the desired position in a virtual structure is defined as x_{di} for the i 'th agent. Control procedure of the multi-agent structure is shown as Algorithm 1.

Remark 2. Although the delay value in the considered problem is fixed, however if the data transmission protocol in the communication network produces time-stamped data packets, the suggested control strategy can be used for the variable delay case. Note that the delay value between different agents can be distinct i.e. τ in (3) can be replaced easily with τ_{ij} without any impact on the performance of the control system.

Augmenting the control inputs of all agents in (3), the overall control law is as follows:

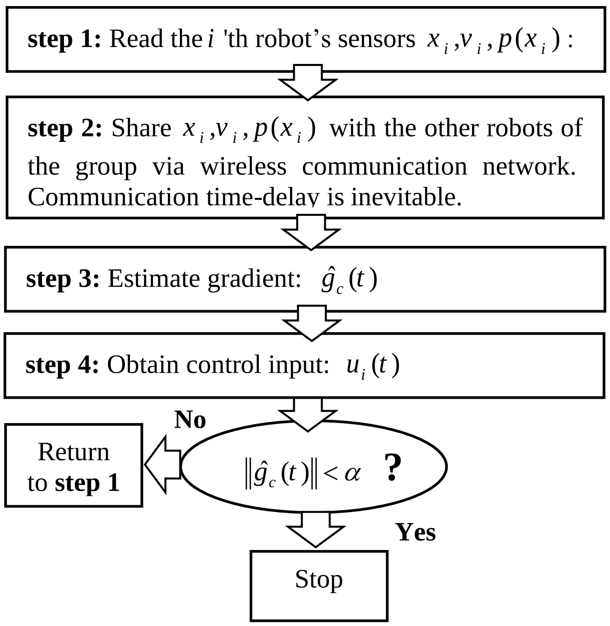
$$\begin{aligned} U(t) = {} & c_0\left(\mathbf{1}_N \otimes \hat{g}_c\right) - c_1 V\left(t\right) \\ & -c_2\left(L \otimes I_n\right)V\left(t-\tau\right) \\ & -c_3\left(L \otimes I_n\right)\left(X\left(t-\tau\right) - X_d\right) \end{aligned} \qquad (4)$$

where $U = \left[u_1\left(t\right), u_2\left(t\right), \ldots, u_N\left(t\right)\right]^T$ represents the control input of all robots, $\mathbf{1}_N$ is a $N \times 1$ vector of which all the entries are equal to one, and $\otimes$ is the Kronecker product, I_n is $n \times n$ identity matrix, in which n represents the dimension of the space, $V\left(t\right) = \left[v_1^T\left(t\right), v_2^T\left(t\right), \ldots, v_N^T\left(t\right)\right]^T$, $V\left(t-\tau\right) = \left[v_1^T\left(t-\tau\right), v_2^T\left(t-\tau\right), \ldots, v_N^T\left(t-\tau\right)\right]^T$, $X\left(t-\tau\right) = \left[x_1^T\left(t-\tau\right), x_2^T\left(t-\tau\right), \ldots, x_N^T\left(t-\tau\right)\right]^T$, and $X_d = \left[x_{d1}^T, x_{d2}^T, \ldots, x_{dN}^T\right]^T$ denotes the velocity vector, delayed velocity vector, delayed position vector,

[DOI: 10.52547/jocee.1.1.85]

and desired positions of all robots respectively. The main result of the paper is presented in Theorem 1.

Algorithm 1.
Source seeking based on the gradient estimation.



Theorem 1. Consider a multi-agent system with the members modelled by (1) and the control law (3); the convergence error of formation center, x_c into the source position, despite of is ultimately bounded x_s communication delay τ , if there exist positive scalars γ satisfying P and positive definite matrix , q_2 , q_1 ,

$$E + \gamma\tau K < 0 \tag{5}$$

Such that $\gamma > \dfrac{\lambda_{\max}(PHH^T P)}{\varepsilon}$ and $q_1 c_1 \geq q_2$, where, E and K are defined as below

$$E = \begin{bmatrix} (G-H)^T P + P(G-H) & PH \\ H^T P & -\gamma I \end{bmatrix},$$

$$K = \begin{bmatrix} (G-H)^T \\ H^T \end{bmatrix} \begin{bmatrix} G-H & H \end{bmatrix} \text{ with}$$

$$G = \begin{pmatrix} 0 & I \\ 0 & -c_1 I \end{pmatrix}, H = \begin{pmatrix} 0 & 0 \\ c_3(J \otimes I) & c_2(J \otimes I) \end{pmatrix}.$$

Where J is created by all positive real part eigenvalues of Laplacian matrix of communication graph.

Proof. We define the new state variable as follows

$$\psi(t) = \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix} = \begin{bmatrix} X(t) - X_d - \mathbf{1} \otimes x_c(t) \\ V(t) - \mathbf{1} \otimes v_c(t) \end{bmatrix}.$$

Calculating time derivative of $\psi(t)$ yields:

$$\begin{aligned} \dot{\psi}_1(t) &= \psi_2(t) \\ \dot{\psi}_2(t) &= -c_3(L \otimes I)\psi_1(t-\tau) \\ &\quad - c_2(L \otimes I)\psi_2(t-\tau) - c_1\psi_2(t). \end{aligned} \tag{6}$$

Based on Lemma 1, we make the following variable transformation:

$$\begin{cases} \mu_1(t) = W^T \psi_1(t) \\ \mu_2(t) = W^T \psi_2(t). \end{cases} \tag{7}$$

According to the structure of ψ_1 and ψ_2 which compare position and speed with the mean value respectively and the structure of $W = \left[\left(\frac{1}{N}\right)\mathbf{1} \quad W'\right]$ as presented in Lemma 1, it can be concluded that the initial conditions $\mu_{11}(0)$ and $\mu_{21}(0)$ are equal to zero if the desired position X_d is defined symmetrically or $\sum_{i=1}^{N} x_{di} = 0$. Since theit can be $\dot{\mu}_{11}(t) = \dot{\mu}_{21}(t) = 0$ concluded that $\mu_{11}(t) = \mu_{21}(t) = 0$. So, regarding variable transformation (7), the equation (6) can be rewritten as follows:

$$\begin{aligned} \dot{\mu}_1(t) &= \begin{pmatrix} \dot{\mu}_{11}(t) \\ \dot{\mu}_{12}(t) \end{pmatrix} = \mu_2(t) = \begin{pmatrix} 0 \\ \mu_{22}(t) \end{pmatrix} \\ \dot{\mu}_2(t) &= \begin{pmatrix} \dot{\mu}_{21}(t) \\ \dot{\mu}_{22}(t) \end{pmatrix} \\ \dot{\mu}_{21}(t) &= 0 \\ \dot{\mu}_{22}(t) &= -c_3(J \otimes I)\mu(t-\tau) \\ &\quad - c_2(J \otimes I)\mu_{22}(t-\tau) - c_1\mu_{22}(t). \end{aligned} \tag{8}$$

Where J is defined in Lemma 1 and is created by all positive real part eigenvalues of L . For brevity, we define $\Omega(t) = \begin{pmatrix} \mu_{12}(t) \\ \mu_{22}(t) \end{pmatrix}$. So we have

$$\begin{aligned} \dot{\Omega}(t) &= G\Omega(t) - H\Omega(t-\tau) \\ G &= \begin{pmatrix} 0 & I \\ 0 & -c_1 I \end{pmatrix}, \\ H &= \begin{pmatrix} 0 & 0 \\ c_3(J \otimes I) & c_2(J \otimes I) \end{pmatrix}. \end{aligned} \tag{9}$$

Now, assume the candidate Lyapunov function below:

[DOI: 10.52547/jocee.1.1.85]

$$V(t) = \tilde{V}(t) + V'(t) \tag{10}$$

in which

$$\tilde{V}(t) = \tilde{V}_1(t) + \tilde{V}_2(t) \tag{11}$$

with

$$\tilde{V}_1(t) = \Omega(t)^T P \Omega(t) \tag{12}$$

$$\tilde{V}_2(t) = \gamma\tau \int_{t-\tau}^{t} (s - t + \tau) \dot{\Omega}(s)^T \dot{\Omega}(s) ds \tag{13}$$

and

$$V'(t) = V_1'(t) + V_2'(t) + V_3'(t) \tag{14}$$

with

$$V_1'(t) = q_0 \left(F(x_s) - F(x_c(t)) \right) \tag{15}$$

$$V_2'(t) = \frac{q_1}{2} v_c(t)^T v_c(t) \tag{16}$$

$$V_3'(t) = q_2 \int_{t-\tau}^{t} v_c^T(s) v_c(s)\, ds \tag{17}$$

Note that V is radially unbounded and positive definite and also consists of two parts, the first part, $\tilde{V}$ ensures that the position of agents converges to the $X_d + \mathbf{1} \otimes x_c$ and the speed of agents converges to the speed of formation center and the second part, V' guarantees that the position of the center converges to the source.

The derivation of $\tilde{V}$ along the system (9) are given by:

$$\dot{\tilde{V}}_1(t) = \dot{\Omega}(t)^T P \Omega(t) + \Omega(t)^T P \dot{\Omega}(t) \tag{18}$$

$$\dot{\tilde{V}}_2(t) = \gamma\tau^2 \dot{\Omega}(t)^T \dot{\Omega}(t) - \gamma\tau \int_{t-\tau}^{t} \dot{\Omega}(s)^T \dot{\Omega}(s) ds \tag{19}$$

Substituting (9) in (18) and (19) and defining new variable $\bar{\Omega}(t) = \Omega(t) - \Omega(t - \tau)$ from lemma 2 we have:

$$\begin{aligned}\dot{\tilde{V}}(t) &\le \Omega(t)^T \left((G-H)^T P + P(G-H) \right) \Omega(t) \\ &+ 2\Omega(t)^T PH \bar{\Omega}(t) - \gamma \bar{\Omega}(t)^T \bar{\Omega}(t) \\ &+ \gamma\tau^2 \left((G-H)\Omega(t) + H\bar{\Omega}(t) \right)^T \left((G-H)\Omega(t) + H\bar{\Omega}(t) \right) \\ &= \left[\Omega(t)^T, \bar{\Omega}(t)^T \right] (E + \gamma\tau K) \begin{bmatrix} \Omega(t) \\ \bar{\Omega}(t) \end{bmatrix}\end{aligned} \tag{20}$$

Where $E = \begin{bmatrix} (G-H)^T P + P(G-H) & PH \\ H^T P & -\gamma I \end{bmatrix}$ and $K = \begin{bmatrix} (G-H)^T \\ H^T \end{bmatrix} \begin{bmatrix} G-H & H \end{bmatrix}$.

It is not difficult to render $(G-H)$ Hurwitz stable by choosing appropriate $c_1, c_2, c_3 > 0$. Then, there is a symmetric and positive definite matrix P such that $(G-H)^T P + P(G-H) < -\varepsilon I$. Note that E is negative definite by the Schur complement theorem if $-\varepsilon I + \frac{1}{\gamma} PHH^T P < 0$ thus we choose $\gamma > \frac{\lambda_{\max}(PHH^T P)}{\varepsilon}$. It follows that $E + \gamma\tau K < 0$ holds when τ is sufficiently close to zero. Thus, there is a positive constant ρ so thatBy . $\dot{\tilde{V}} < -\rho \|\Omega(t)\|$ Lyapunov-Krasovskii theorem [21], $X(t) \to X_d + \mathbf{1} \otimes x_c(t)$ and $V(t) \to \mathbf{1} \otimes v_c(t)$. In addition, the derivation of $v_c(t)$ is given by:

$x_c(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t) = \frac{1}{N} (\mathbf{1} \otimes I_n) X$ then $\dot{x}_c(t) = \frac{1}{N} (\mathbf{1} \otimes I_n) V = v_c(t)$. Because of the assumption 4 we know that $(\mathbf{1}^T \otimes I)(L \otimes I) = 0$, then

$$\begin{aligned}\dot{v}_c(t) &= -c_1 \frac{1}{N} (\mathbf{1} \otimes I_n) V(t) + c_0 \hat{g}_c \\ &= -c_1 v_c(t) + c_0 \hat{g}_c\end{aligned} \tag{21}$$

also,

$$\dot{V}_1'(t) = -q_0 g_c^T v_c(t) \tag{22}$$

$$\dot{V}_2'(t) = -q_1 c_1 v_c^T(t) v_c(t) + q_1 c_0 v_c^T(t) \hat{g}_c. \tag{23}$$

Assume that the gradient estimation $\hat{g}_c(t)$ has a bounded error, $\| \hat{g}_c(t) - g_c(t) \| \le e$; where, e is a positive constant, using Cauchy–Schwarz inequality and choosing $q_0 = q_1 c_0$, the sum of V_1' and V_2' can be expressed as follows:

[DOI: 10.52547/jocee.1.1.85]

$$\begin{aligned}
&\dot{V}_1'(t)+\dot{V}_2'(t)= \\
&-q_1c_1v_c^T(t)v_c(t)+2q_1c_0v_c^T(t)\hat{g}_c-q_0g_c^Tv_c(t) \\
&= -q_1c_1\|v_c(t)\|_2^2+q_0v_c^T(t)(\hat{g}_c-g_c) \\
&\le -q_1c_1\|v_c(t)\|_2^2+q_0\|v_c^T(t)(\hat{g}_c-g_c)\|_2 \\
&\le -q_1c_1\|v_c(t)\|_2^2+q_0\|v_c(t)\|_2\|\hat{g}_c-g_c\|_2 \\
&\le -q_1c_1\|v_c(t)\|_2^2+q_0e\|v_c(t)\|_2
\end{aligned} \tag{24}$$

and

$$\dot{V}_3'(t)=q_2v_c^T(t)v_c(t)-q_2v_c^T(t-\tau)v_c(t-\tau) \tag{25}$$

Combining *(24)* and *(25)* gives:

$$\begin{aligned}
\dot{V}'(t)\le{}& eq_0\|v_c(t)\|_2-(q_1c_1-q_2)\|v_c(t)\|_2^2 \\
&-q_2\|v_c(t-\tau)\|_2^2.
\end{aligned} \tag{26}$$

Now we can write the derivation form of $V(t)$ as follows

$$\begin{aligned}
\dot{V}(t)\le{}&\left[\Omega(t)^T,\bar{\Omega}(t)^T\right](E+\gamma\tau K)\begin{bmatrix}\Omega(t)\\ \bar{\Omega}(t)\end{bmatrix} \\
&+eq_0\|v_c(t)\|_2-(q_1c_1-q_2)\|v_c(t)\|_2^2 \\
&-q_2\|v_c(t-\tau)\|_2^2.
\end{aligned} \tag{27}$$

Choosing $q_2\le q_1c_1$, regardless of first and last negative terms of (27) we have:

$$\dot{V}(t)\le eq_0\|v_c(t)\|_2-(q_1c_1-q_2)\|v_c(t)\|_2^2. \tag{28}$$

The right-hand side of (28) is not negative definite because, in the proximity of origin, the positive linear term $q_0\|v_c(t)\|_2$ dominates the negative quadratic term $-(q_1c_1-q_2)\|v_c(t)\|_2^2$. However, $\dot{V}(t)$ is negative outside the set $\|v_c(t)\|_2\ge\dfrac{eq_0}{q_1c_1-q_2}$. By choosing $q_0,q_2\to 0$ and $q_1\to\infty$ we can expand the negative definite range of .$\dot{V}(t)$

Outside the region $\|v_c(t)\|_2\ge\dfrac{eq_0}{q_1c_1-q_2}$, the right side of (27) is negative and a function of $\|v_c(t)\|_2$; Thus, it can be concluded that $\|v_c(t)\|_2$ reduces down to $\dfrac{eq_0}{q_1c_1-q_2}$. Although this result is conservative because first and last negative terms of (27) has been omitted, and the result will definitely be better by considering these terms. From $\dot{\tilde{V}}<0$, we conclude that $X\to X_d+\mathbf{1}\otimes x_c$ and $V\to\mathbf{1}\otimes v_c$.

By employing Algorithm 1, agents stop at x_c^*, where $\hat{g}_c(x_c^*)\to 0$; regarding the concept of ultimate boundedness, we show that the convergence error is bounded in what follows. The Taylor expansion of $F(x)$ at x_c^* and $\bar{x}$ between x and x_c^* yields

$$\begin{aligned}
F(x)={}&\frac{1}{2}(x-x_c^*)^T H(\bar{x})(x-x_c^*) \\
&+\nabla^T F(x_c^*)(x-x_c^*)+F(x_c^*).
\end{aligned}$$

For $x=x_s$, this equation leads to

$$\begin{aligned}
F(x_s)-F(x_c^*)={}&\frac{1}{2}(x_s-x_c^*)^T H(\bar{x})(x_s-x_c^*) \\
&+\nabla^T F(x_c^*)(x_s-x_c^*).
\end{aligned}$$

Provided that $\|\hat{g}_c(t)-g_c(t)\|\le e$, and from Assumption 3 we have

$$\begin{aligned}
&0<F(x_s)-F(x_c^*)\le-\frac{\kappa_2}{2}\left\|x_s-x_c^*\right\|^2+\left\|\nabla^T F(x_c^*)\right\|\left\|x_s-x_c^*\right\| \\
&=-\frac{\kappa_2}{2}\left\|x_s-x_c^*\right\|^2+\left\|\nabla^T F(x_c^*)-\hat{g}_c(x_c^*)\right\|\left\|x_s-x_c^*\right\| \\
&\le-\frac{\kappa_2}{2}\left\|x_s-x_c^*\right\|^2+\mathrm{e}\left\|x_s-x_c^*\right\|.
\end{aligned}$$

This leads to $\left\|x_s-x_c^*\right\|\le\dfrac{2e}{\kappa_2}$.■

IV. Cooperative Gradient Estimatiom

In this section, the distributed gradient estimator using delayed information is derived to estimate $\hat{g}_c(t)$ the field gradient at the formation center $x_c(t)$ at time t . Each agent can access its own local objective function, $E_i(a,b)$ which is determined by its sensor measurements and data sent form the neighbors, including field value and position. It is worth noting the spatial distribution of the field parameters is different, so the group members sense distinct values for the same parameter, therefore cooperation is made to enhance the quality of gradient estimation in the center of the formation. Moreover, the main advantage of the distributed methods is to increase the reliability of the group missions.

The agents in the system cooperate to achieve the solution of the following optimization problem to obtain the gradient value at the formation center.

[DOI: 10.52547/jocee.1.1.85]

$$\min_{a,b}\ \mathrm{E(a,b)}=\frac{1}{N}\sum_{i=1}^{N}E_i\left(a,b\right) \tag{29}$$

Where $E_i\left(a,b\right)=\sum_{j\in N(i)}\left(y_j^i-\hat{y}_j^i\right)^2$ is the sum of the squared difference between measurements, y_j^i and estimations, $\hat{y}_j^i$ of the field value, $F(x)$ by the j 'th robot from the point of view of the i 'th robot. We assume that the estimated field value to be linear $\hat{y}_j^i=ax_j^i+b$, where a and b are two constants and x_j^i is the position of j 'th robot from the point of view of the i 'th robot. For more accurate estimation, instantaneous information of the robot and the most recent received information of the neighbors are used, i.e. x_j^i and y_j^i are as $x_i(t)$ and $y_i(t)$ for $j=i$, and $x_j\left(t-\tau\right)$, and $y_j\left(t-\tau\right)$ for $j\neq i$ respectively.

Remark 3. Each E_i is differentiable and convex over R^n . Thus, E is convex because of the convexity of E_i , $i=1,\ldots,N$ and it guarantees a unique optimal solution for
(29).

As noted in [22], optimization problem

(29) is equivalent to the following problem on $R^{N\times n}$:

$$\min_{a,b}\ \tilde{\mathrm{E}}\mathrm{(a,b)}=\frac{1}{N}\sum_{i=1}^{N}E_i\left(a^i,b^i\right) \qquad s.t.\quad \mathbf{L}\boldsymbol{\omega}=0_{N\times n} \tag{30}$$

, where $\mathbf{L}=L\otimes I_n$, in whichthe Laplace represents L matrix of the graph G and $\boldsymbol{\omega}=\left[\omega^{1^T}\quad\cdots\omega^{N^T}\right]^T$, are the b^i and a^i , i=1,...,N for $\omega_i=\left[a^{i^T}\quad b^{i^T}\right]^T$ parameters of estimation from the point of view of the i 'th robot.

It is notable that the $\left(a,b\right)$ is the common state of local objective functions in problem

(29) and a,b is decoupled in problem through adding an equality constraint (30), so that each local objective function depends on the state of an individual agent only $\left(a^i,b^i\right)$. The optimal solution of each local objective function can be written as follows

$$\min_{a^i,b^i}\ E_i\left(a^i,b^i\right) \tag{31}$$

$$\nabla E_i(a^i,b^i)=0 \tag{32}$$

$$\begin{pmatrix}a^{i*}\\ b^{i*}\end{pmatrix}=\begin{pmatrix}\sum_{j\in\{N(i),i\}}x_j^{i\,2} & \sum_{j\in\{N(i),i\}}x_j^i\\ \sum_{j\in\{N(i),i\}}x_j^{i\,2} & \sum_{j\in\{N(i),i\}}1\end{pmatrix}^{-1}\begin{pmatrix}\sum_{j\in\{N(i),i\}}x_j^iy_j^i\\ \sum_{j\in\{N(i),i\}}y_j^i\end{pmatrix} \tag{33}$$

$$\hat{g}_{ci}=\left(I_N\quad \mathbf{0}_{N\times 1}\right)\begin{pmatrix}a^{i*}\\ b^{i*}\end{pmatrix}. \tag{34}$$

Therefore, the gradient vector of the formation center, $\hat{g}_{ci}$ from the point of view of i th agent is calculated using (34) which is used in the control law (3). In fact, the equation (34) gives an optimal estimation of $\hat{g}_{ci}$ in the sense of least squares.

Theorem 2. Let $\mathbf{x}^*=\mathbf{1}\otimes x^*$, in which x^* satisfies $\hat{g}_c(\mathbf{x}^*)=0$; $\left[\mathbf{x}^*+X_d,\mathbf{0}\right]^T$ is an equilibrium of multi-agent system composed of robots modelled by (1) with the control law (3) and $\left[\hat{g}_{c1}{}^T\quad \hat{g}_{c2}{}^T\quad \ldots\quad \hat{g}_{cN}{}^T\right]^T$ is an optimum solution of

(29) and vice versa.

Proof. Suppose that the equilibrium point of multi-agent system (1) is $\left[\mathbf{x}^*+X_d,\mathbf{0}\right]^T$, then we have

$$\mathbf{0}=V \tag{35}$$

$$\begin{aligned}\mathbf{0}&=c_0\left[\hat{g}_{c1}{}^T\quad \hat{g}_{c2}{}^T\quad \ldots\quad \hat{g}_{cN}{}^T\right]^T-c_1V\\ &\quad-c_2\left(L\otimes I_n\right)V-c_3\left(L\otimes I_n\right)\left(X-X_d\right).\end{aligned} \tag{36}$$

Where, $\hat{g}_{c1},\hat{g}_{c2},\ldots,\hat{g}_{cN}$ are the optimal solution of each local objective function. Substituting (35) in (36) we have

$$\begin{aligned}\mathbf{0}&=c_0\left[\hat{g}_{c1}{}^T\quad \hat{g}_{c2}{}^T\quad \ldots\quad \hat{g}_{cN}{}^T\right]^T\\ &\quad-c_3\left(L\otimes I_n\right)\left(X-X_d\right).\end{aligned} \tag{37}$$

Multiplying (37) by $\left(\mathbf{1}_N\otimes I_n\right)^T$ from the left, from Remark 3 we obtain that $X-X_d=\mathbf{x}^*=\mathbf{1}\otimes x^*$, where $x^*\in R^n$ is a constant vector and $\sum_{i=1}^{N}\hat{g}_{ci}=\sum_{i=1}^{N}\nabla E_i\left(a^{i*},b^{i*}\right)=\nabla\tilde{\mathrm{E}}(\mathrm{a}^*,\mathrm{b}^*)=0$. Then the optimality condition $\nabla E\left(a^*,b^*\right)=0$ along with (30) is satisfied, which means that $\left(a^*,b^*\right)$ is the optimum solution of

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(29) and satisfies equality constraint $\mathbf{L\omega} = 0_{N \times n}$. The conversion is easy through substituting $\left(\mathbf{x}^* + X_d, \mathbf{0}\right)$ into (35) and (36).■

Remark 4. Although, in problem (29), the information of all the agents is incorporated to obtain the gradient estimation, our solution (33) in Theorem 2 only uses the neighbors' information to calculate the gradient estimation in each robot.

V. SIMULATION

Here, a simulation example is given to validate the obtained results; then, to provide evidence for the real-world applicability of the developed method, the simulation scenario is implemented in Webots® software which is a specialized tool to prototype robots.

The source seeking problem with a team of five agents is taken into account. Information of the agents is exchanged by communication network with constant delay $\tau = 0.36$ s; communication topology is modelled with a weight-balanced and directed connected graph as illustrated in Fig. 1, in which the Laplacian matrix is as follows:

$$L = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

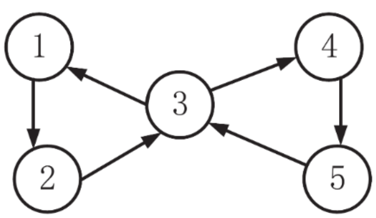

Fig. 1. Communication topology

As [4], we choose $F(x) = -\|x\|^2$, so that the maximum value is at the origin where the source is located. The parameters in control input (3) are designed as $c_0 = 0.2$, $c_1 = 0.8$, $c_2 = 0.8$, and $c_3 = 0.8$. With these parameters, the LMIs in Theorem 1 are feasible. The initial values for the positions of the agents 1-5 are chosen as [8 9 7.5],[8 -9 7],[-8 9 6.5],[-8 -9 8], and [9 8 6] respectively. In addition, the initial values for the velocities are chosen to be zero. The x_{di} which is the chosen position of the i th robot in the desired formation is considered as x_{d1}=[-1 1 0] , x_{d2}=[-1 -1 0] , x_{d3}=[0 0 0], x_{d4}=[1 1 0] , and x_{d5}=[1 -1 0] for the robots 1 to 5.

The simulation scenario was first implemented in Matlab®. As seen in Fig. 2-Fig. 5, the robots are able to move to the desired position around the source with a slight error and stop there. Namely, all the robots converge to the desired position around the source.

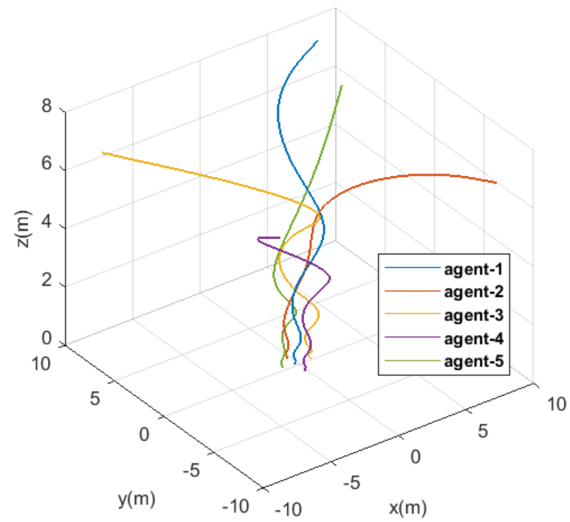

Fig. 2. Three-dimensional view of the agents` positions

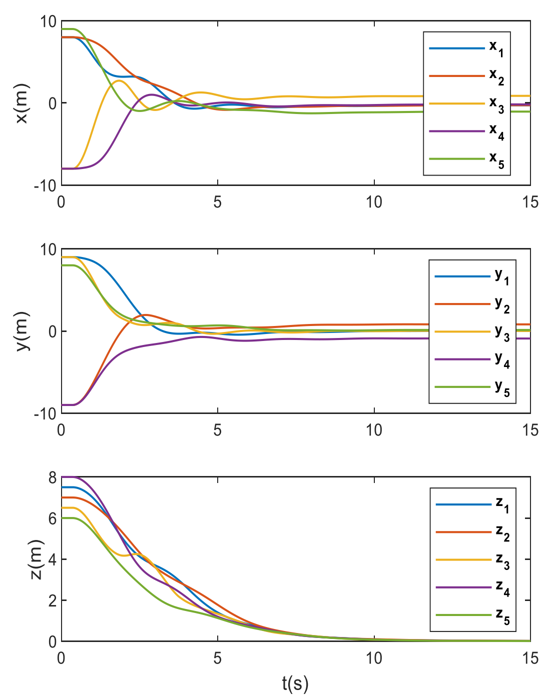

Fig. 3. Positions of the agents

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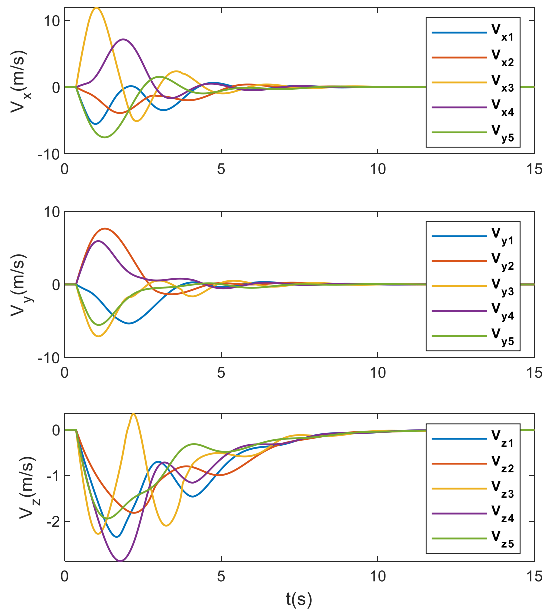

Fig. 4. Velocities of the agents

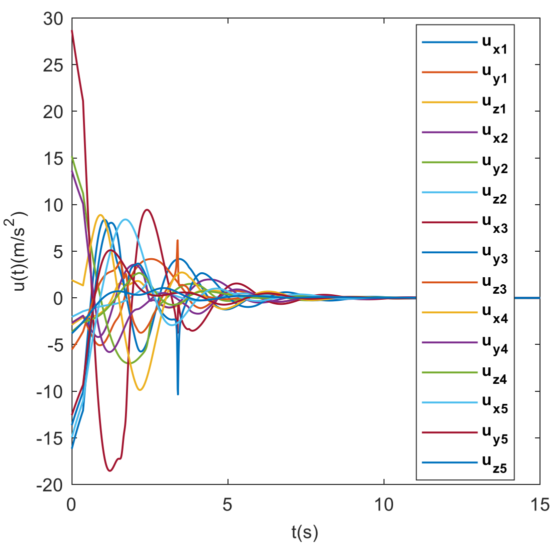

Fig. 5. Control inputs of the agents

To check the effect of the distributed gradient estimator, the following are presented:

1) For different delays, the convergence time is reported in Table I for the following conditions:

- when the exact value of the formation center's gradient, g_c is available.
- The gradient is estimated from the proposed method, $\hat{g}_c$.

2) Assuming that the desired formation for the agents in the previous example is zero, i.e. $x_d = \mathbf{0}$, the norm of error between the exact and estimated value of the gradient in the center of robot's position ($\| g_c - \hat{g}_{c_i} \|_2$) is depicted in Fig. 6 .

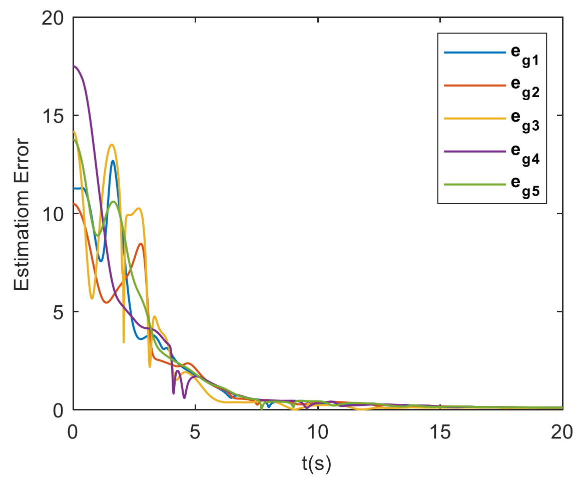

Fig. 6. Norm of error between the exact and estimated value of the gradient in the center of robot's position

A light seeking scenario is simulated in Webots® using the proposed control scheme and the actual applicability of the introduced scheme is shown. A mobile robot that is Differential-driven wheeled and has two step motors were utilized. Navigation sensors including a GPS, two light sensors, a 3-D accelerometer, and gyroscope were employed on the robot and radio module was used for communication. The mobile robots were moved by altering the relative angular velocities of driving wheels. It was assumed that the body of the robot was stable and there was no sliding in the motion. The wheel rotation happened only in one axis. Thus, the navigation was performed by altering the speed of either side of the robot.

Table I. Comparision of convergence time by applying different delays

Delay (millisecond)		***100***	***200***	***300***	***400***	***500***	***560***
convergence time	**With exact value of gradient (g_c)**	12	15	17	20	60	Diverged
	With estimation of gradient ($\hat{g}_c$)	15	18	22	24	72	Diverged

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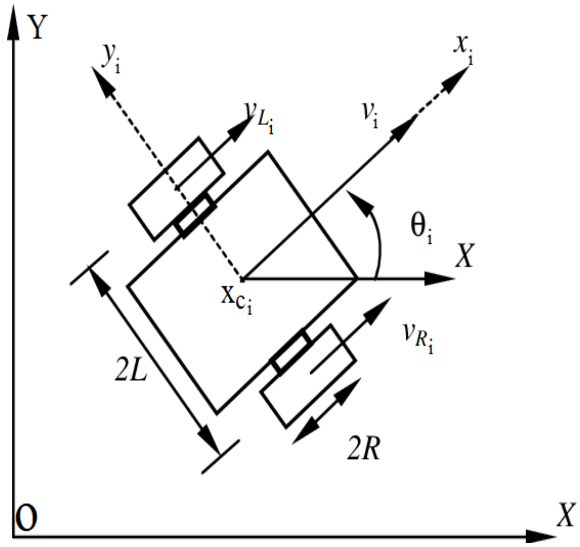

Fig. 7. Schematics of the considered mobile robots

Fig. 7 illustrates a schematic of the differential drive mobile robot where $\{O, X, Y\}$ represents the global coordinate and $\{x_{c_i}, x_i, y_i\}$ represents the local coordinate fixed to the robot with its center x_{c_i}, R as the radius of the wheels and $2L$ represents the gap between two driving wheels, v_{L_i} and v_{R_i} are the velocity of the driving wheels on the left and right, the angle θ_i represents the robot orientation, and x_{c_i} is the center of the i 'th mobile robot.

The motion of the robot is in two-dimensional space, $R = 0.04m, L = 0.06m$, the communication delay is $\tau = 0.36$ seconds ,and all the parameters and conditions of the simulation are considered as in the previous scenario, except the initial values for the positions of the agents 1-5 that are chosen respectively as $[4\ \ 4\ \ 0],[4\ \ 3\ 0],[0\ \ 3\ \ 0],[-4\ \ 0\ \ 0],[-4\ \ 4\ \ 0]$. The desired formation is considered as follows, $x_{d1}=[0\ \ 0\ \ 0]$ $x_{d3}=[1\ \ 0\ \ 0]$, $x_{d2}=[0\ \ 1\ \ 0]$, $x_{d4}=[0\ \ -1\ \ 0]$, $x_{d5}=[-1\ \ 0\ \ 0]$.

As illustrated in Fig. 11, there is a complete agreement between the Webots® outcome animation and the outcomes of the pertinent simulation made using the simple dynamic models in Matlab® (Fig. 8-Fig. 10). It is worth noting that the Webots® program can be easily ported to the actual robots to do an real-world experimental test.

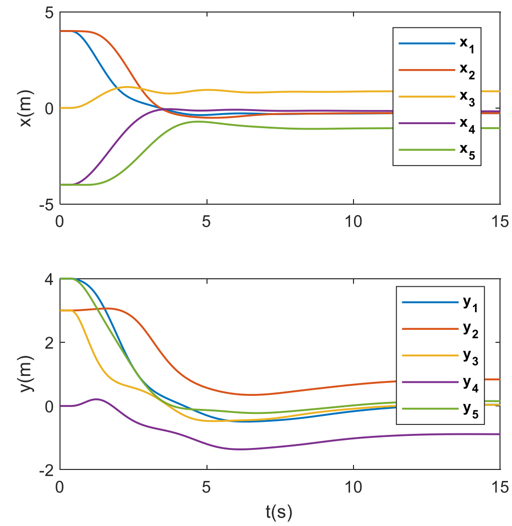

Fig. 8. Robots' positions with respect to time during movement

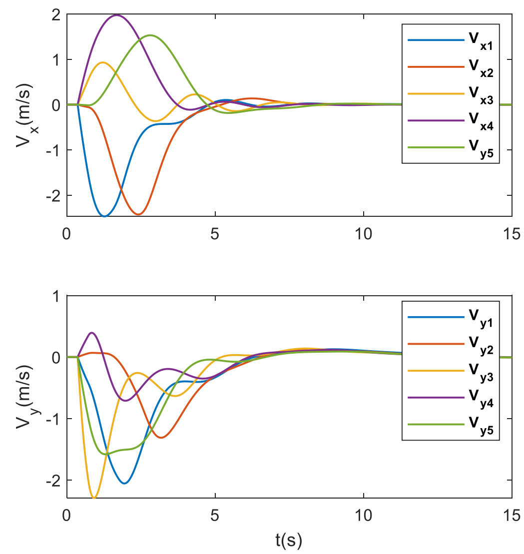

Fig. 9. Robots' velocities with respect to time during movement

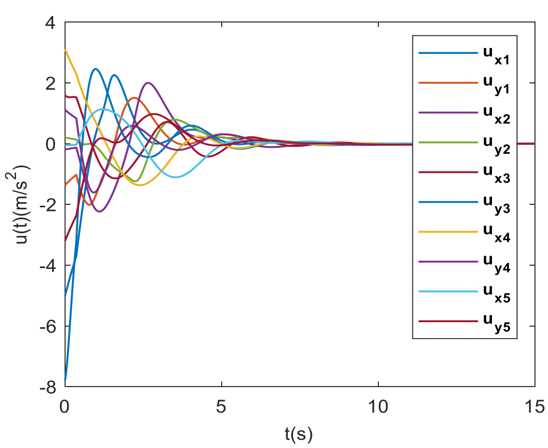

Fig. 10. Control inputs of the agents

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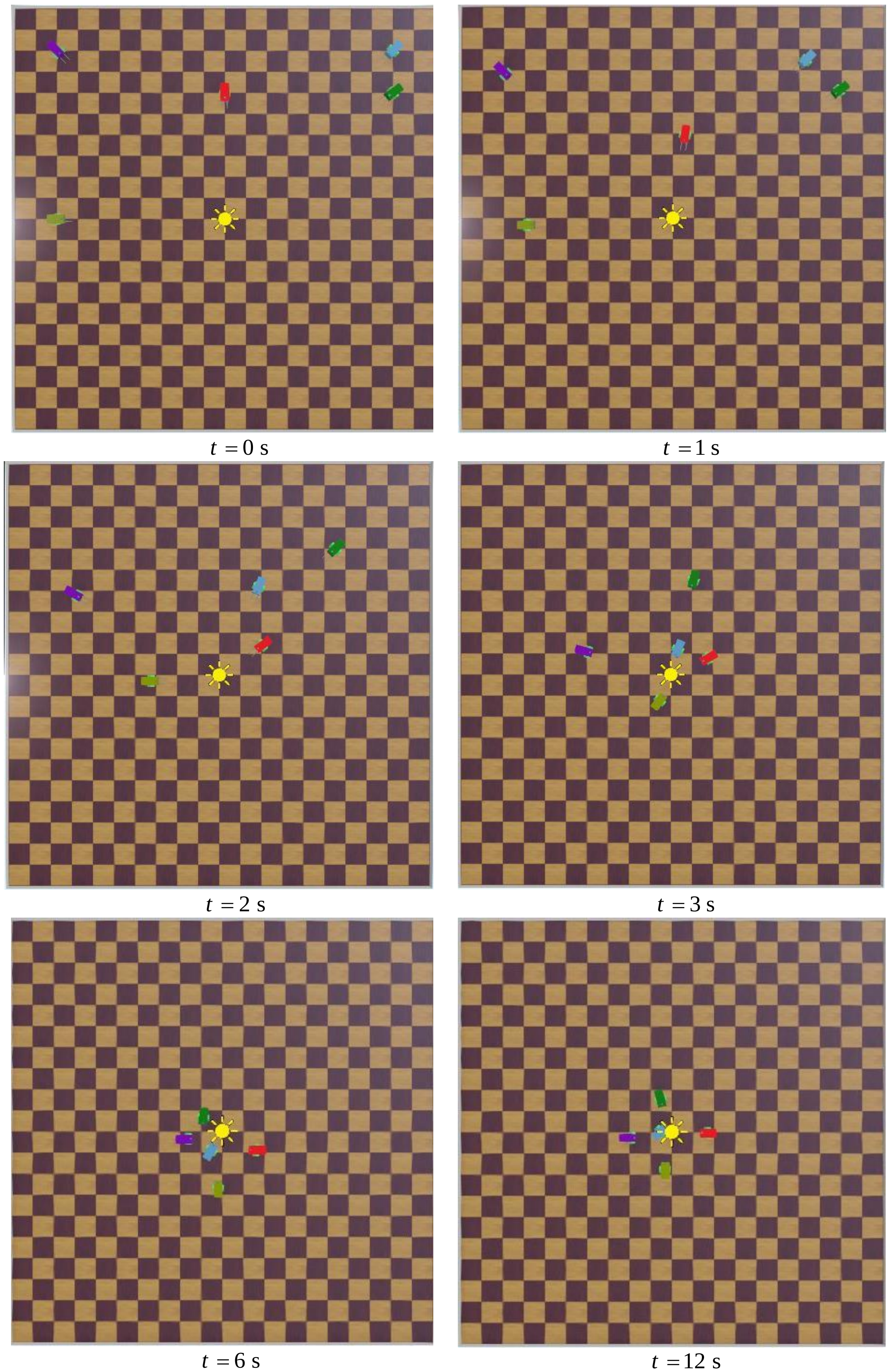

Fig. 11. Snapshots of the robots' motion in Webots®

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VI. CONCLUTION

In this paper, distributed control of multi-agent systems was addressed for source seeking with communication time-delays. According to the Lyapunov-Krasovskii theory for time-delay systems, sufficient condition was derived in the form of LMIs to tune the parameters of the proposed control law. Furthermore, a novel cooperative gradient estimator with delayed information was proposed by solving a distributed optimization problem. Applicability and efficiency of the proposed controller was studied through simulations. Many avenues are open for further investigations; for instance, adding conditions to the suggested source seeking algorithm to prevent agents from colliding with each other and obstacles, or investigating source seeking problem in switching topologies with communication delays.

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