Distributed formation tracking control of a multi-quadrotor system based on sliding mode and a rate bounded PID controller in the presence of internal perturbation and external disturbance

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***Abstract*— In this paper, a distributed consensus problem with path tracking problem for multiple quadrotors with bounded disturbances under a directed topology is solved. At first, dynamic model of a single quadrotor is divided into two subsystems; fully-actuated and under-actuated subsystem. Some coupling terms between the two subsystems are regarded as internal perturbations. Bounded external disturbances are considered in this paper too and are entered to the quadrotors. Rate bounded PID controller and sliding mode controller are considered for fully-actuated subsystem of the quadrotors, for altitude and yaw angle of the quadrotors respectively. For under-actuated subsystems of the quadrotors, a distributed control protocol based on sliding mode control is proposed to guarantee that the consensus is achieved and the desired path is tracked by the group with bounded errors. These errors are only associated with external disturbances. Analysis based on Lyapunov theory and different simulations show the stability and effectiveness of the proposed control system.**

***Keywords*: Multi-agent system, Quadrotor, sliding mode controller, distributed control protocol.**

I. INTRODUCTION

This paper is focused on the cooperative control of a multi-quadrotor system with a particular communication topology. One of the basic issues in the field of multi-agent dynamical systems on networks is design and development of distributed control protocols to ensure consensus between the considered states among group agents; and formation control is another issue which researchers are focused on it nowadays.

In recent years, due to the enormous advantages and abilities, the quadrotors have attracted the more attention of companies, institutes and research centers and so daily progress is noticeable on their facilities and flying.

In [1], by simplifying the quadrotor's dynamic equations and using a nonlinear control method and a leader-follower structure, the formation flight problem was solved. In 2011, for regional coverage by several quadrotors, distributed control protocols were used based on location optimization techniques and a leader-follower structure, formation and consensus problem solved for several quadrotors by simplified dynamic equations [2]. In [3], with the linearization of the quadrotor's dynamic equations and with several other simplification assumptions, the problem of formation control of several quadrotors with a leader-follower structure, along with collision avoidance was investigated. In [4], formation control for a group of quadrotors was also discussed with a simplified model, and a different distributed control protocol based on dynamic surface control was presented using the leader-follower structure. In [5], the model of the quadrotor was considered with the assumption that the Euler angles are small. In that research, a specific control protocol is provided to formation control and path tracking of several quadrotors. This control protocol uses the inverse

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dynamical for outer control loop as well as the backstepping control for inner control loop. Several quadrotors reach a target point by creating a special formation and using the dynamic surface control in [6]. Path tracking and obstacle avoidance, as well as protocol design based on fuzzy logic and fuzzy cellular decomposition algorithm, have been used to obtain the shortest path to the target point in [7]. For designing protocol, simplified dynamic equations of the quadrotors were used. Formation control of several quadrotors using sliding mode control along with disturbances has been subject of paper [8]. It is worth noting that external disturbances are not observed in that research and that only internal disturbances are not neglected. Simulation and experimental test of formation path tracking of several quadrotors, despite the change in the communication topology and with linearized equations, was investigated in [9]. Guero et al used two control loops to control their quadrotors group [10]. The first loop ensures the stability of each quadrotor based on the Lyapunov theory, and the other loop helps the group to track a specified path and reach to the desired point. The consensus problem was brought up for several quadrotors using the feedback linearization control and a local feedback control in [11]. An event-triggered-based optimal distributed formation control algorithm was proposed for a multi-quadrotor system in [12]. A simplified dynamic model of the UAV was used in the mentioned research. Using the linearized model of quadrotors for their formation control, as well as the collision avoidance, has been the subject of researches in [13-14]. A time-varying anti-disturbance formation problem was investigated for a group of quadrotor aircrafts with time-varying uncertainties and directed interaction topology in [15]. A leader-follower structure was considered for solving the formation problem; however, the leader was not considered as an agent of the system. In [16], the problem of consensus and formation control for several quadrotors was investigated by introducing a visual algorithm. The model of the quadrotors was simplified and leader-follower structure was used in that research.

As mentioned above, consensus and formation control problems for several quadrotors have been solved by various types of control theories in various works. However, several simplifying assumptions were found in them for describing dynamical equations of quadrotors, e.g., ignoring the aerodynamic forces applied to the quadrotors, or assuming small Euler angles. Leader-follower structure was used in many researches. When a failure is occurred to leader, this structure cannot be overcome to this issue and mission will be failed. In addition, all the mentioned researches developed a distributed formation control for a multi-quadrotor system directly and not for a general multi-agent system. Therefore, the main aim of this article is designing a new distributed formation control for a class of nonlinear multi-agent system which these agents have inherent nonlinear under-actuated dynamical model. A multi quadrotor system can be considered as a special case of this multi-agent system. In this research, simplifying assumptions in modeling of the quadrotors are reduced, e.g., the aerodynamic forces applied to the quadrotors are not ignored and so on. In addition, a virtual structure based distributed formation control is proposed for an under-actuated nonlinear multi-agent system and is developed for a multi-quadrotor system.

The dynamical model of quadrotors is divided into two subsystems, fully-actuated subsystem and under-actuated subsystem. The coupled terms between the two subsystems in this study, contrary to previous researches, are considered as internal disturbances applied to quadrotors. A bounded rate PID controller and a sliding mode control is considered for the fully-actuated subsystem. For the under-actuated subsystem, a distributed protocol control based on sliding mode control is proposed. The proposed protocol guarantees the consensus of the desired states as well as tracking an arbitrary path by the group with a small error due to external disturbances. Furthermore, the aerodynamic forces applied to the quadrotors are not neglected and are included in the dynamical equations of the quadrotors.

In the second section, the dynamics of quadrotors is presented in the presence of external disturbances. The third section addresses the design of a distributed control protocol for quadrotors and analyzes its stability. Section 4 illustrates the performance of the proposed protocol using the simulated example. Finally, in the fifth section, the conclusions are presented.

II. Quadrotors dynamic

In this section, a mathematical model describing the dynamics of a quadrature is presented. As previously mentioned, the quadrotors dynamic equations are divided into two subsystems.

It is assumed that a group of n quadrotors with the following dynamical equations is available for the i_{th} quadrotor [17].

$$\ddot{x}_i = (\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i)u_{1i} - K_{1i}\frac{\dot{x}_i}{m_i} + \delta_{xi} \tag{1}$$

$$\ddot{y}_i = (\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i)u_{1i} - K_{2i}\frac{\dot{y}_i}{m_i} + \delta_{yi}$$
$$\ddot{z}_i = -g + (\cos\phi_i \cos\theta_i)u_{1i} - K_{3i}\frac{\dot{z}_i}{m_i}$$
$$\ddot{\varphi}_i = u_{2i} - l_i K_{4i}\frac{\dot{\varphi}_i}{I_{1i}}$$
$$\ddot{\theta}_i = u_{3i} - l_i K_{5i}\frac{\dot{\theta}_i}{I_{2i}}$$
$$\ddot{\psi}_i = u_{4i} - K_{6i}\frac{\dot{\psi}_i}{I_{3i}}$$

where (x_i, y_i, z_i) are the center of gravity coordinates and $(\varphi_i, \theta_i, \psi_i)$ are Euler angles of the i_{th} quadrotor. g is the earth's gravity acceleration, l_i is the distance between motors and center of gravity and m_i is the total mass of the i_{th} quadrotor. I_{pi} $(p = 1,2,3)$ are inertia moments relative to the body coordinates. K_{pi} $(p = 1,2,\dots,6)$ are constants of the aerodynamic forces entered to the i_{th} quadrotor. δ_{xi} and δ_{yi} are external disturbances entered to the i_{th} quadrotor. u_{qi} $(q = 1,2,3,4)$ are virtual control inputs defined as (2).

$$\begin{aligned} u_{1i} &= (F_{1i} + F_{2i} + F_{3i} + F_{4i})/m_i \\ u_{2i} &= l_i(-F_{1i} - F_{2i} + F_{3i} + F_{4i})/I_{xi} \\ u_{3i} &= l_i(-F_{1i} + F_{2i} + F_{3i} - F_{4i})/I_{yi} \\ u_{4i} &= C(F_{1i} - F_{2i} + F_{3i} - F_{4i})/I_{zi} \end{aligned} \quad (2)$$

where F_{pi} $(p = 1,2,3,4)$ is the thrust force produced by p_{th} motor of the i_{th} quadrotor. C is the coefficient proportional to the length of the blade. Dynamical model of the i_{th} quadrotor (1), divided into a fully-actuated subsystem (3) and an under-actuated subsystem (4).

$$\begin{bmatrix}\ddot{z}_i \\ \ddot{\psi}_i\end{bmatrix} = \begin{bmatrix}u_{1i}(\cos\phi_i \cos\theta_i) - g \\ u_{4i}\end{bmatrix} + \begin{bmatrix}-K_{3i}\,\dot{z}_i/m_i \\ -K_{6i}\,\dot{\psi}_i/I_{3i}\end{bmatrix} \quad (3)$$

$$\begin{bmatrix}\ddot{x}_i \\ \ddot{y}_i\end{bmatrix} = u_{1i}\begin{bmatrix}\cos\psi_i & \sin\psi_i \\ \sin\psi_i & -\cos\psi_i\end{bmatrix}\begin{bmatrix}\cos\phi_i \sin\theta_i \\ \sin\phi_i\end{bmatrix} + \begin{bmatrix}-K_{1i}\,\dot{x}_i/m_i \\ -K_{2i}\,\dot{y}_i/m_i\end{bmatrix} + \begin{bmatrix}\delta_{xi} \\ \delta_{yi}\end{bmatrix} \quad (4)$$

$$\begin{bmatrix}\ddot{\varphi}_i \\ \ddot{\theta}_i\end{bmatrix} = \begin{bmatrix}u_{2i} \\ u_{3i}\end{bmatrix} + \begin{bmatrix}-l_i K_{4i}\,\dot{\varphi}_i/I_{1i} \\ -l_i K_{5i}\,\dot{\theta}_i/I_{2i}\end{bmatrix}$$

Since the drag force is very small at low velocities, we can consider terms with a drag coefficient in the above equations in the form of small disturbances, which will be explained below.

III. DISTRIBUTED CONTROL PROTOCOL

For fully-actuated subsystem (3), a bounded rate PID controller and a sliding mode control can be designed [18]. Desired control input for z_i is

$$u_{1di} = \frac{k_{z1i}(z_{di} - z_i) + k_{z2i}\int(z_{di} - z_i)dt}{\cos\phi_i \cos\theta_i} + \frac{k_{z3i}(\dot{z}_{di} - \dot{z}_i)}{\cos\phi_i \cos\theta_i} + \frac{g}{\cos\phi_i \cos\theta_i} \quad (5)$$

where k_{z1i}, k_{z2i} and k_{z3i} are control parameters and z_{di} is the desired altitude of the i_{th} quadrotor. A bounded rate controller can be converged u_{1i} to u_{1di}.

$$\dot{u}_{1i} = k_i \times \operatorname{sat}\left(\frac{k_{0i}\int(u_{1di} - u_{1i})dt + k_{1i}(u_{1di} - u_{1i})}{\varepsilon_i}\right) \quad (6)$$

where $\operatorname{sat}(.)$ is saturation function. k_i, k_{0i}, k_{1i} and ε_i are other control parameters. The initial conditions are chosen so that $u_{1i}(0) = u_{1di}(0)$. Thus,

$$|\dot{u}_{1i}| \le k_i \quad (7)$$

On the other hand, for quadrotors rotation angle, a sliding mode controller is used to make ψ_i to converge to its desired value ψ_{di} in the presence of disturbances.

$$u_{4i} = -c_{\psi i}\dot{\psi}_i - M_{\psi i}\operatorname{sign}(s_{\psi i}) - k_{\psi i}s_{\psi i} \quad (8)$$

where $c_{\psi i}, M_{\psi i}$ and $k_{\psi i}$ are control parameters which are all positive. $s_{\psi i} = c_{\psi i}(\psi_i - \psi_{di}) + \dot{\psi}_i$ is stable sliding surface which are designed for ψ_i.

Before developing the controller for the under-actuated subsystem (4), below under-actuated system is considered.

It is assumed that a group of n agents with the following dynamical equations for the i_{th} agent is available.

$$\begin{aligned} \dot{x}_{1i} &= x_{2i} + d_{1i} \\ \dot{x}_{2i} &= f(x_{3i}) + d_{2i} + \nabla_i \\ \dot{x}_{3i} &= x_{4i} \\ \dot{x}_{4i} &= u_i + d_{3i} \end{aligned} \quad (9)$$

where $x_{pi} \epsilon R^m$ $(p = 1,2,3,4)$ are state variables of the i_{th} agent. $u_i \epsilon R^m$ is input control. $d_{1i}, d_{2i}, d_{3i} \epsilon R^m$ are internal disturbances and ∇_i are external disturbances exerted to the i_{th} quadrotor. f is a smooth vector function. The virtual leader for these n agents is defined with same dynamic $x_{p0} \epsilon R^m$ $(p = 1,2,3,4)$. State variables of the x_{p0} are always known and are presented in the form of the desired path. The following assumptions are considered for controller design.

Assumption 1: $f(0) = 0$

Assumption 2: Jacobian matrix of $f(x)$ shown with $J(f)$ are always positive definite.

Assumption 3: Maximum of the values row summation of the matrix $J(f)$ is bounded, so that $\|J(f(x_{3i}))\|_\infty < \beta_i$ where β_i is a positive constant.

Assumption 4: The finite norm of the difference between internal disturbances are bounded function of the state differences. In the sense that

$$\sum_{j=1}^{n} a_{ij}\|d_{1j} - d_{1i}\| < \bar{d}_1 \sum_{j=1}^{n} a_{ij}\|x_{1j} - x_{1i}\|$$

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$$\sum_{j=1}^{n} a_{ij}\|d_{2j}-d_{2i}\| < \bar{d}_2 \sum_{j=1}^{n} a_{ij}\|x_{2j}-x_{2i}\|$$
$$\sum_{j=1}^{n} a_{ij}\left\|J\left(f(x_{3j})\right)d_{3j}-J(f(x_{3i}))d_{3i}\right\| < \beta_i\bar{d}_3\sum_{j=1}^{n} a_{ij}\|x_{4j}-x_{4i}\|$$

where $\beta_i(i=1,2,\dots,n)=\beta$. Moreover, for the difference of external disturbances, we have

$$\sum_{j=1}^{n} a_{ij}\|\nabla_j-\nabla_i\| < \bar{\nabla}\sum_{j=1}^{n} a_{ij}\|\delta_j-\delta_i\|$$

To stabilize all states of the agents, as well as path tracking of the desired path along with the formation control of the group agents some errors are defined as

$$\begin{aligned} e_{1i} &= \sum_{j=1}^{n} a_{ij}(x_{1j}-x_{1i}-\Delta_{ij}) \\ e_{2i} &= \sum_{j=1}^{n} a_{ij}(x_{2j}-x_{2i}) \\ e_{3i} &= \sum_{j=1}^{n} a_{ij}\{f(x_{3j})-f(x_{3i})\} \\ e_{4i} &= \sum_{j=1}^{n} a_{ij}\{J(f(x_{3j}))x_{4j}-J(f(x_{3i}))x_{4i}\} \end{aligned} \tag{10}$$

where a_{ij} is the i_{th} row of the J_{th} from communication graph Laplacian matrix [19]. A sliding variable for the i_{th} agent is defined as

$$S_i = C_{1i}e_{1i}+C_{2i}e_{2i}+C_{3i}e_{3i}+e_{4i} \tag{11}$$

where C_{pi} $(p=1,2,3)$ are control parameters and it is explained in more detail on choosing them in future. The distributed control protocol is proposed as

$$\begin{aligned} u_i &= \frac{\mathrm{inv}\big(J(f(x_{3i}))\big)}{\sum_{j=1}^{n} a_{ij}}\Bigg\{C_{1i}\sum_{j=1}^{n} a_{ij}(x_{2j}-x_{2i}) \\ &+ C_{2i}\sum_{j=1}^{n} a_{ij}\big(f(x_{3j})-f(x_{3i})\big) \\ &+ C_{3i}\sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))x_{4j} - J(f(x_{3i}))x_{4i}\big) \\ &+ \sum_{j=1}^{n} a_{ij}\left(\frac{d}{dt}\Big(J\big(f(x_{3j})\big)\Big)x_{4j} - \frac{d}{dt}\big(J(f(x_{3i}))\big)x_{4i}\right) \\ &+ \sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))u_j\big) + M_i\mathrm{sign}(S_i) \\ &+ \nu_i S_i\Bigg\} \end{aligned} \tag{12}$$

where $\nu_i > 0$ and M_i defines as below:

$$\begin{aligned} M_i &= C_{1i}\bar{d}_1\sum_{j=1}^{n} a_{ij}\|x_{1j}-x_{1i}\| \\ &+ C_{2i}\bar{d}_2\sum_{j=1}^{n} a_{ij}\|x_{2j}-x_{2i}\| \\ &+ C_{2i}\bar{\nabla}\sum_{j=1}^{n} a_{ij}\|\delta_j-\delta_i\| \\ &+ \beta\bar{d}_3\sum_{j=1}^{n} a_{ij}\|x_{4j}-x_{4i}\| \\ &+ \rho_i \end{aligned} \tag{13}$$

where $M_i > 0$ and $\rho_i > 0$. It is shown that this protocol can guarantee consensus of states, as well as formation control of the group and well path tracking with bounded error,

Theorem: Consider the dynamical model expressed in (9) together with the assumptions 1-4. Using of the distributed control protocol (12), all the errors defined in (10) converge to zero.

Proof: Regardless of disturbances, the equivalent control u_{ieq}, which makes the sliding variable of the i_{th} agent to converge the sliding surface $S_i=0$, is proposed as

$$\begin{aligned} u_{ieq} &= \frac{\mathrm{inv}\big(J(f(x_{3i}))\big)}{\sum_{j=1}^{n} a_{ij}}\Bigg\{C_{1i}\sum_{j=1}^{n} a_{ij}(x_{2j}-x_{2i}) \\ &+ C_{2i}\sum_{j=1}^{n} a_{ij}\big(f(x_{3j})-f(x_{3i})\big) \\ &+ C_{3i}\sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))x_{4j} - J(f(x_{3i}))x_{4i}\big) \\ &+ \sum_{j=1}^{n} a_{ij}\left(\frac{d}{dt}\Big(J\big(f(x_{3j})\big)\Big)x_{4j} - \frac{d}{dt}\big(J(f(x_{3i}))\big)x_{4i}\right) \\ &+ \sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))u_j\big)\Bigg\} \end{aligned} \tag{14}$$

Now, by taking disturbances into account, the control input u_i must be designed in such a way that guarantees the convergence of S_i to the $S_i=0$ surface.

$$u_i = u_{ieq}+u_{isw} \tag{15}$$

where u_{isw} is switching term of the sliding mode control of the i_{th} agent. The candidate ofLyapunov function is $V=\sum_{i=1}^{n} V_i$ where V_i is chosen as

$$V_i = \frac{1}{2}S_i^T S_i \tag{16}$$

Time derivative of this function is

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$$
\begin{aligned}
\dot{V}_i &= S_i^T \dot{S}_i \\
&= S_i^T \Big\{ C_{1i} \sum_{j=1}^{n} a_{ij}(x_{2j} - x_{2i}) \\
&+ C_{1i} \sum_{j=1}^{n} a_{ij}(d_{1j} - d_{1i}) \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}\big(f(x_{3j}) - f(x_{3i})\big) \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}(d_{2j} - d_{2i}) \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}(\nabla_j - \nabla_i) \\
&+ C_{3i} \sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))x_{4j} \\
&- J(f(x_{3i}))x_{4i}\big) \\
&+ \sum_{j=1}^{n} a_{ij}\left(\frac{d}{dt}\big(J(f(x_{3j}))\big)x_{4j} \right.\\
&\left. - \frac{d}{dt}\big(J(f(x_{3i}))\big)x_{4i}\right) \\
&+ \sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))u_j - J(f(x_{3i}))u_i\big) \\
&+ \sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))d_{3j} \\
&- J(f(x_{3i}))d_{3i}\big)\Big\}
\end{aligned}
\tag{17}
$$

By substituting (15) into (17), time derivative of the Lyapunov function obtained as follows.

$$
\begin{aligned}
\dot{V}_i = S_i^T \dot{S}_i = S_i^T \Big\{ & C_{1i} \sum_{j=1}^{n} a_{ij}(d_{1j} - d_{1i}) \\
& + C_{2i} \sum_{j=1}^{n} a_{ij}(d_{2j} - d_{2i}) \\
& + C_{2i} \sum_{j=1}^{n} a_{ij}(\nabla_j - \nabla_i) \\
& - \sum_{j=1}^{n} a_{ij}\big(J(f(x_{3i}))\big)u_{isw} \\
& + \sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))d_{3j} \\
& - J(f(x_{3i}))d_{3i}\big)\Big\}
\end{aligned}
\tag{18}
$$

u_{isw} is chosen such as bellow.

$$
u_{isw} = \frac{\mathrm{inv}\big(J(f(x_{3i}))\big)}{\sum_{j=1}^{n} a_{ij}} \{M_i \mathrm{sign}(S_i) + \nu_i S_i\} \tag{19}
$$

Now, time derivative of the Lyapunov function candidate is

$$
\begin{aligned}
&\dot{V}_i \\
&= S_i^T \Big\{ C_{1i} \sum_{j=1}^{n} a_{ij}(d_{1j} - d_{1i}) \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}(d_{2j} - d_{2i}) \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}(\nabla_j - \nabla_i) \\
&+ \sum_{j=1}^{n} a_{ij}\big(J(f(x_{3j}))d_{3j} - J(f(x_{3i}))d_{3i}\big) \\
&- M_i \mathrm{sign}(S_i) - \nu_i S_i \Big\} \\
&= -\nu_i \|S_i\|^2 \\
&+ S_i^T \Big\{ C_{1i} \sum_{j=1}^{n} a_{ij}(d_{1j} - d_{1i}) \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}(d_{2j} - d_{2i}) \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}(\nabla_j - \nabla_i) \\
&+ \sum_{j=1}^{n} a_{ij}\Big(J\big(f(x_{3j})\big)d_{3j} \\
&- J\big(f(x_{3i})\big)d_{3i}\Big) - M_i \mathrm{sign}(S_i)\Big\} \\
&< -\nu_i \|S_i\|^2 \\
&+ S_i^T \Big\{ -M_i \mathrm{sign}(S_i) \\
&+ C_{1i} \sum_{j=1}^{n} a_{ij}\|d_{1j} - d_{1i}\| \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}\|d_{2j} - d_{2i}\| \\
&+ C_{2i} \sum_{j=1}^{n} a_{ij}\|\nabla_j - \nabla_i\| \\
&+ \sum_{j=1}^{n} a_{ij}\|J(f(x_{3j}))d_{3j} \\
&- J(f(x_{3i}))d_{3i}\|\Big\} \\
&< -\nu_i \|S_i\|^2 \\
&+ \Big\{ -M_i + C_{1i}\bar{d}_1 \sum_{j=1}^{n} a_{ij}\|x_{1j} - x_{1i}\| \\
&+ C_{2i}\bar{d}_2 \sum_{j=1}^{n} a_{ij}\|x_{2j} - x_{2i}\| \\
&+ C_{2i}\bar{\nabla} \sum_{j=1}^{n} a_{ij}\|\delta_j - \delta_i\| \\
&+ \beta \bar{d}_3 \sum_{j=1}^{n} a_{ij}\|x_{4j} - x_{4i}\| \Big\} \|S_i\|_1
\end{aligned}
\tag{20}
$$

By substituting (13) into the last part of (20), inequality for the time derivative of V_i changes to

$$
\dot{V}_i < -\nu_i \|S_i\|^2 - \rho_i \|S_i\|_1 \le 0 \tag{21}
$$

Therefore, the sliding variable (11) of the under-actuated i_{th} agent (9) by using distributed control protocol (12) converges to sliding surface $S_i = 0$ in finite time and will remain on it. On the sliding surface $S_i = 0$,

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it can be shown that with a correct choice of $C_{pi}(p = 1,2,3)$, all errors (10) are bounded. On the sliding surface $S_i = 0$, the general system of the i_{th} agent is reduced to

$$\begin{bmatrix}\dot e_{1i}\\ \dot e_{2i}\\ \dot e_{3i}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -C_{1i} & -C_{2i} & -C_{3i}\end{bmatrix}\begin{bmatrix}e_{1i}\\ e_{2i}\\ e_{3i}\end{bmatrix} + \begin{bmatrix}\sum_{j=1}^{n} a_{ij}\left(d_{1j} - d_{1i}\right)\\ \sum_{j=1}^{n} a_{ij}\left(d_{2j} - d_{2i}\right) + \sum_{j=1}^{n} a_{ij}\left(\nabla_j - \nabla_i\right)\\ 0\end{bmatrix} = A_i E_i + D_i \tag{22}$$

where $E_i = [e_{1i} \quad e_{2i} \quad e_{3i}]^T$, $\sum_{j=1}^{n} a_{ij}\left\|d_{1j} - d_{1i}\right\| < \bar d_1\|e_{1i}\| + \bar d_1\left\|\sum_{j=1}^{n} a_{ij}\Delta_{ij}\right\|$ and $\sum_{j=1}^{n} a_{ij}\left\|d_{2j} - d_{2i}\right\| + \sum_{j=1}^{n} a_{ij}\left(\nabla_j - \nabla_i\right) < \bar d_2\|e_{2i}\| + \bar\nabla\left\|\sum_{j=1}^{n} a_{ij}(\delta_j - \delta_i)\right\|$.

(22) is a perturbed system with both of vanishing perturbation section and no vanishing perturbation section. It can be proved that, the origin is global exponentially stable for the vanishing perturbation section and is stable with bounded error for the no vanishing perturbation section if A_i is Hurwitz and $\max(\bar d_1, \bar d_2) < \lambda_{\min}(-A_i)$, where $\lambda_{\min}(-A_i)$ is the real part of the minimum left eigenvalues of $-A_i$ [18]. Therefore, e_{1i} and e_{2i} are bounded in a small interval and e_{3i} converges to zero for any initial condition, if $C_{pi}(p = 1,2,3)$ are chosen such that A_i be Hurwitz and $\max(\bar d_1, \bar d_2) < \lambda_{\min}(-A_i)$. Finally, due to the fact that $S_i = C_{1i}e_{1i} + C_{2i}e_{2i} + C_{3i}e_{3i} + e_{4i} = 0$, e_{4i} is also bounded.

■

Now it's time to use this presented sliding mode control for n quadrotors. Since the design of the controllers for the fully-actuated subsystem is independent of the controller design for the under-actuated subsystem of the quadrotor, a sliding mode controller can be designed for ψ_i which aims to achieve fast convergence of ψ_i to its desired value. Therefore, by this design, it may be said that after a finite time, ψ_i reaches its desired value, and since then it is almost constant. In addition, by using the controller (5), after a finite time, u_{1i}'s converge to their stable state values, $g/(\cos\phi_i\cos\theta_i)$.

$$\left|u_{1i} - \frac{g}{(\cos\phi_i\cos\theta_i)}\right| \le \eta_i \le \frac{\eta_i}{|\cos\phi_i\cos\theta_i|} \tag{23}$$

where $\eta_i > 0$ is a small constant. Therefore, the low boundary of u_{1i} is determined by

$$|u_{1i}| \ge \left|\frac{g - \eta_i}{(\cos\phi_i\cos\theta_i)}\right| \tag{25}$$

The below coordinate transformation matrix is required for developing controllers of the fully-actuated subsystem of the quadrotors.

$$T_i = u_{1i}\begin{bmatrix}\cos\psi_i & \sin\psi_i\\ \sin\psi_i & -\cos\psi_i\end{bmatrix} = u_{1i}\hat T(\psi_i) \tag{26}$$

Since ψ_i is approximately invariant over time, $\hat T(\psi_i)$ can be assumed constant and represented by $\hat T$. Determinant of the matrix T_i is $-{u_{1i}}^2 < 0$ if $u_{1i} \ne 0$. Therefore, since in the general conditions u_{1i} is the power of the four motors of the i_{th} quadrotor in the direction z of the body coordinates and to overcome to the force of gravity of the earth, the matrix T_i is always nonsingular. the following conversions are considered.

$$\begin{aligned} x_{1i} &= {T_i}^{-1}\begin{bmatrix}x_i\\ y_i\end{bmatrix}\\ x_{2i} &= {T_i}^{-1}\begin{bmatrix}\dot x_i\\ \dot y_i\end{bmatrix}\\ x_{3i} &= \begin{bmatrix}\varphi_i\\ \theta_i\end{bmatrix}\\ x_{4i} &= \begin{bmatrix}\dot\varphi_i\\ \dot\theta_i\end{bmatrix}\end{aligned} \tag{27}$$

By deriving from the new states of the i_{th} quadrotor, we have

$$\begin{aligned} \dot x_{1i} &= {T_i}^{-1}\begin{bmatrix}\dot x_i\\ \dot y_i\end{bmatrix} + \frac{d}{dt}\left(\frac{1}{u_{1i}}\right)\hat T^{-1}\begin{bmatrix}x_i\\ y_i\end{bmatrix} = x_{2i} - \frac{\dot u_{1i}}{u_{1i}}x_{1i}\\ \dot x_{2i} &= {T_i}^{-1}\begin{bmatrix}\ddot x_i\\ \ddot y_i\end{bmatrix} + \frac{d}{dt}\left(\frac{1}{u_{1i}}\right)\hat T^{-1}\begin{bmatrix}\dot x_i\\ \dot y_i\end{bmatrix}\\ &= \begin{bmatrix}\sin\varphi_i\cos\theta_i\\ \sin\theta_i\end{bmatrix} - \hat T^{-1}\begin{bmatrix}K_{1i}/m_i & 0\\ 0 & K_{2i}/m_i\end{bmatrix}\hat T x_{2i} - \frac{\hat T^{-1}}{u_{1i}}\begin{bmatrix}\delta_{xi}\\ \delta_{yi}\end{bmatrix} - \frac{\dot u_{1i}}{u_{1i}}x_{2i}\\ \dot x_{3i} &= x_{4i}\\ \dot x_{4i} &= \begin{bmatrix}u_{2i}\\ u_{3i}\end{bmatrix} + \begin{bmatrix}-l_iK_{4i}/I_{1i} & 0\\ 0 & -l_iK_{5i}/I_{2i}\end{bmatrix}x_{4i}\end{aligned} \tag{28}$$

Given

$$\begin{aligned} f(x_{3i}) &= \begin{bmatrix}\sin\varphi_i\cos\theta_i\\ \sin\theta_i\end{bmatrix}\\ u_i &= \begin{bmatrix}u_{2i}\\ u_{3i}\end{bmatrix}\\ d_{1i} &= -\frac{\dot u_{1i}}{u_{1i}}x_{1i}\\ d_{2i} &= -\frac{\dot u_{1i}}{u_{1i}}x_{2i} - \hat T^{-1}\begin{bmatrix}K_{1i}/m_i & 0\\ 0 & K_{2i}/m_i\end{bmatrix}\hat T x_{2i}\\ d_{3i} &= \begin{bmatrix}-l_iK_{4i}/I_{1i} & 0\\ 0 & -l_iK_{5i}/I_{2i}\end{bmatrix}x_{4i}\\ \nabla_i &= -\frac{\hat T^{-1}}{u_{1i}}\begin{bmatrix}\delta_{xi}\\ \delta_{yi}\end{bmatrix}\end{aligned} \tag{29}$$

the under-actuated subsystem of the i_{th} quadrotor (4) can be transformed into (9). Given the relations (7) and (24), the following bounded functions are obtained.

$$\|d_{1j}-d_{1i}\| = \left\|-\frac{\dot{u}_{1j}}{u_{1j}}x_{1j}+\frac{\dot{u}_{1i}}{u_{1i}}x_{1i}\right\| \le \left\|\left|\frac{(\cos\phi_i\cos\theta_i)k_i}{g-\eta_i}\right|x_{1i} - \left|\frac{(\cos\phi_j\cos\theta_j)k_j}{g-\eta_j}\right|x_{1j}\right\| \quad (30)$$

$$\sum_{j=1}^{n} a_{ij}\|d_{1j}-d_{1i}\| < \frac{k}{|g-\eta|}\sum_{j=1}^{n} a_{ij}\|x_{1j}-x_{1i}\|$$

This is obtained with the condition that all k_i and all η_i $(i=1,2,\dots,n)$ are respectively k and η.

$$\sum_{j=1}^{n} a_{ij}\|d_{2j}-d_{2i}\| \le \left\|\left(\left|\frac{(\cos\phi_i\cos\theta_i)k_i}{g-\eta_i}\right| + \max(K_{1i}/m_i, K_{2i}/m_i)\right)x_{2i} - \left(\left|\frac{(\cos\phi_j\cos\theta_j)k_j}{g-\eta_j}\right| + \max(K_{1j}/m_i, K_{2j}/m_j)\right)x_{2j}\right\| < \left(\frac{k}{|g-\eta|}+\max(K_{1i}/m_i, K_{2i}/m_i)\right)\sum_{j=1}^{n} a_{ij}\|x_{2j}-x_{2i}\| \quad (31)$$

On the other hand,

$$\|J(f(x_{3j}))d_{3j}-J(f(x_{3i}))d_{3i}\| \le \left\|\left|J\left(f(x_{3j})\right)\right|d_{3j} - |J(f(x_{3i}))|d_{3i}\right\| = \left\|J\left(f(x_{3j})\right)\right\|_{\infty}\|d_{3j}-d_{3i}\| < \beta\|d_{3j}-d_{3i}\| \quad (32)$$

in addition

$$\sum_{j=1}^{n} a_{ij}\|d_{3j}-d_{3i}\| < \max(-l_iK_{4i}/I_{1i}, -l_iK_{5i}/I_{2i})\sum_{j=1}^{n} a_{ij}\|x_{3j}-x_{3i}\| \quad (33)$$

$$\sum_{j=1}^{n} a_{ij}\|J(f(x_{3j}))d_{3j}-J(f(x_{3i}))d_{3i}\| < \beta\bar{d}_3\sum_{j=1}^{n} a_{ij}\|x_{4j}-x_{4i}\|$$

Since $\|\hat{T}^{-1}\| \le 2$ and by using the relation (25), one can write

$$\|\nabla_j-\nabla_i\| = \left\|-\frac{\hat{T}^{-1}}{u_{1j}}\begin{bmatrix}\delta_{xj}\\ \delta_{yj}\end{bmatrix}+\frac{\hat{T}^{-1}}{u_{1i}}\begin{bmatrix}\delta_{xi}\\ \delta_{yi}\end{bmatrix}\right\| \le \left\|\frac{\hat{T}^{-1}}{u_{1i}}\right\|\|\delta_j-\delta_i\| < \frac{2}{|g-\eta|}\|\delta_j-\delta_i\| \quad (34)$$

$$\sum_{j=1}^{n} a_{ij}\|\nabla_j-\nabla_i\| < \frac{2}{|g-\eta|}\sum_{j=1}^{n} a_{ij}\|\delta_j-\delta_i\|$$

Consequently, constants in assumption 4 are defined as

$$\begin{aligned}\bar{d}_1 &= \frac{k}{|g-\eta|}\\ \bar{d}_2 &= \left(\frac{k}{|g-\eta|}+\max(K_{1i}/m_i, K_{2i}/m_i)\right)\\ \bar{d}_3 &= \max(-l_iK_{4i}/I_{1i}, -l_iK_{5i}/I_{2i})\\ \bar{\nabla} &= \frac{2}{|g-\eta|}\end{aligned} \quad (35)$$

All these constants are positive. Therefore, the assumption 4 is satisfied. From (29) we can find that $f(0)=0$, hence, assumption 1 is also satisfied. The Jacobin matrix $f(x_{3i})$ is defined as

$$J(f) = \frac{\partial f}{\partial x_{3i}} = \begin{bmatrix}\cos\varphi_i\cos\theta_i & -\sin\varphi_i\sin\theta_i\\ 0 & \cos\theta_i\end{bmatrix} \quad (36)$$

It should be noted that if $(\varphi_i,\theta_i)\in(-\pi/2,\pi/2)\times(-\pi/2,\pi/2)$, then $\cos\varphi_i\cos\theta_i>0$, as well as the determinant of $J(f)$ is $\cos\varphi_i\cos^2\theta_i>0$. Therefore, $J(f)$ is positive and invertible. In addition, the maximal sum of the absolute value of the matrix rows of $J(f)$ is finite.

$$\|J(f)\|_{\infty} < 2 \quad (37)$$

Therefore, the assumptions 2 and 3 are also satisfied.

In short, the presented distributed sliding mode controller (16) with PID controllers (5) with a bounded rate (6) for quadrotors height as well as sliding mode control for yaw angles (8), gives the group ability to make a desired formation with a small amount of error while obtaining and tracking a desirable path from the virtual leader. When e_{1i} is bounded to a small interval, the group formation is obtained by Δ_{ij}, and the desired path is tracked by the group with a small error. When e_{2i} is bounded, the speed of the quadrotors reaches a consensus with a small error. Furthermore, when e_{3i} and e_{4i} converge to a small interval, the quadrotors attitude,

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reach a consensus.

IV. SIMULATION RESULTS

The main parameters, as well as the initial conditions of the quadrotors, as well as the control parameters, are simulated based on Table 1.

The desired values, as well as the desired path of the virtual leader, as well as external disturbances entered to the quadrotors, used in the simulated example, are in the form

$$\begin{aligned}&\{x_r, y_r, z_r, \psi_r\}^T = \{\sin(t), \cos(t), t, 1\}^T\\&\{\delta_{x1}, \delta_{x2}, \delta_{x3}, \delta_{x4}\} = \varpi\\&\times\{\sin(0.1t), \cos(0.2t), 0.5\sin(t), 0.7\cos(t)\}\\&\{\delta_{y1}, \delta_{y2}, \delta_{y3}, \delta_{y4}\} = \varpi\\&\times\{0.5, 0.8, \sin(0.5t), 0.1\cos(t)\}\end{aligned} \tag{38}$$

where ϖ is a repeating square wave that starts at time $t = 0$, with a width of 1 and also a height of 1. Disturbances are presented in Figure 2. Figure 3 shows the paths that the quadrotors are tracked. It is obvious that the presented control protocol makes the quadrotors to track their own desired paths and makes the group to keep its desired formation during the mission after about four seconds. Figures 4 and 5 show acceptable performance of the presented protocol to solve the consensus problem in the velocity and attitudes of the quadrotors, respectively. By using this presented protocol, robustness against the disturbances is achieved as show in figures 3-5. Figure 6 shows obviously, performance of the designed protocol in formation control problem.

Remark 1. $\tanh(.)$ is a continuous approximation of the sign function which is used instead of that to avoid of chattering in simulations.

V. CONCLUSION

In this paper, a distributed consensus problem along with path tracking for a group of quadrotors was evaluated using a directional topology and in the presence of bounded internal and external disturbances. At first, the dynamical model of the quadrotors divided into two subsystems, fully-actuated subsystem and under-actuated subsystem. Some coupled clauses between the two subsystems as well as the drag forces entered the quadrotors introduced as internal disturbances to the under-actuated subsystem. In addition to internal disturbances, certain external disturbances also applied to this system. For the fully-actuated subsystem, two different controllers, a bounded rate PID controller and a sliding mode control were used. These controllers were able to control the fully-actuated subsystem of the quadrotors. For the under-actuated subsystem of the quadrotors, a specially distributed control protocol was developed and presented based on sliding mode control. The proposed distributed protocol solved the consensus problem as well as desired path tracking by the group, which is obtained by using a virtual leader, with a small error resulting from external disturbances. The simulations also showed the effectiveness and efficiency of the proposed control protocol.

TABLE I
SIMULATION PARAMETERS

Constant	Value
$K_{1i} = K_{2i} = K_{3i}$	0.01
$K_{4i} = K_{5i} = K_{6i}$	0.012
$I_{1i} = I_{2i}$	1.25
I_{3i}	2.5
$\{k_{z1i}, k_{z2i}, k_{z3i}\}$	$\{10,5,10\}$
$\{k_{0i}, k_{1i}, k_i, \varepsilon_i\}$	$\{3,5,5,0.1\}$
$\{c_{\psi i}, M_{\psi i}, k_{\psi i}\}$	$\{1,5 \times {}^{2}/_{\pi}, 0.1\}$
$\{C_{1i}, C_{2i}, C_{3i}, \rho_i, \nu_i\}$	$\{40,45,6,1,0.1\}$
$\{\beta, \eta\}$	$\{2,1\}$
$\{x_1(0), x_2(0), x_3(0), x_4(0)\}$	$\{2,1,-0.5,0.25\}$
$\{y_1(0), y_2(0), y_3(0), y_4(0)\}$	$\{-2,0.5,-0.5,1.25\}$
$\{z_1(0), z_2(0), z_3(0), z_4(0)\}$	$\{0,0,0,0\}$
$\{m_i \quad l_i \quad g\}$	$\{2 \quad 0.2 \quad 9.81\}$
$\Delta_{12}, \Delta_{13}, \Delta_{24}, \Delta_{34}$	$\begin{Bmatrix}1\\0\end{Bmatrix}, \begin{Bmatrix}0\\-1\end{Bmatrix}, \begin{Bmatrix}0\\-1\end{Bmatrix}, \begin{Bmatrix}1\\0\end{Bmatrix}$

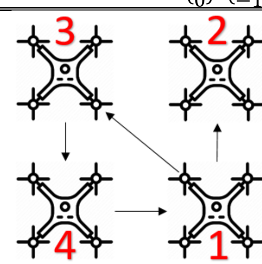

Figure 1. Communication graph

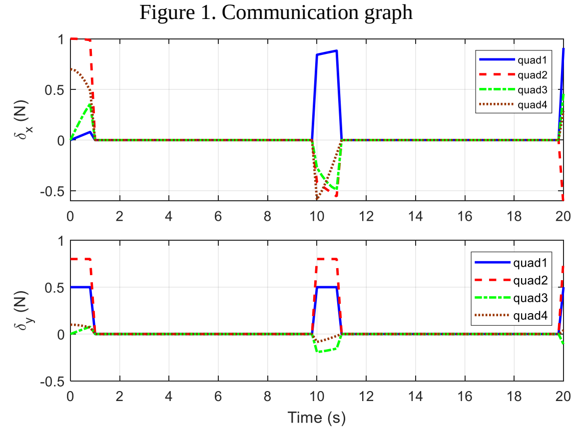

Figure 2. External disturbances exerted to the quadrotors

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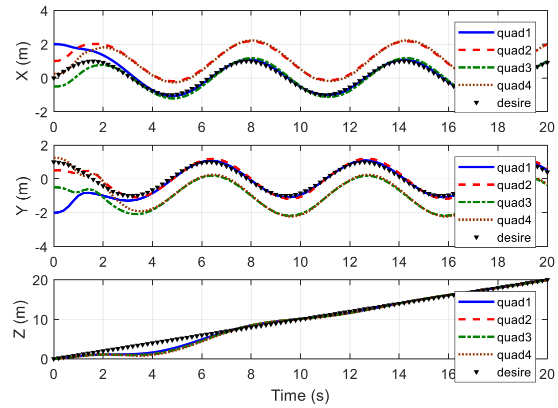

Figure 3. Tracked path by the quadrotors

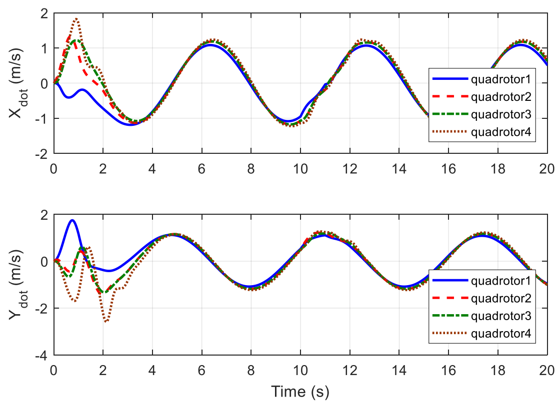

Figure 4. Consensus of the quadrotors velocity

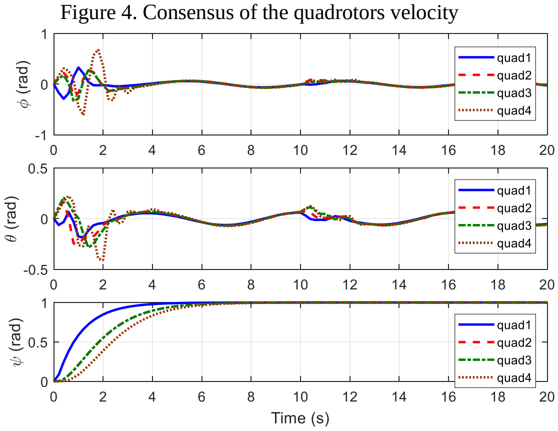

Figure 5. Euler angles of the quadrotors during the path

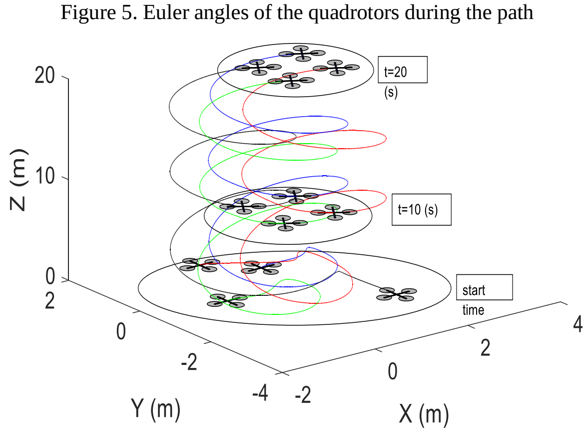

Figure 6. Tracked formation of the quadrotors

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